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Enclosure 1

OUTAGE-OPTIMAL TRANSMISSION IN MULTIUSER-MIMO KRONECKER CHANNELS

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ABSTRACT

In this work, we look at single user and multiuser Multiple-Input Multiple-Output (MIMO) beamforming networks with Channel Distribution Information (CDI). CDI does not need to be updated every time the channel changes, but only when the statistics of the channel change. Thus, feedback in the network is significantly reduced when compared to CSI schemes. With statistical information, we can only guarantee quality of service for a certain probability of outage in the network. The optimal beamformers and an optimal power control algorithm to minimize the power cost function in the network are presented for the well-known Kronecker model.

Index Terms— covariance feedback, beamformers, multiuser MIMO, outage probability, Channel Distribution Information

1. INTRODUCTION

Multiuser multiple-input multiple-output (MU-MIMO) networks have generated much research interest in recent years. Ideally, to achieve maximum throughput and reliability, all the nodes need to have perfect Channel State Information (CSI) of all the links in the MU-MIMO network. With this knowledge, utilizing intelligent beamforming, interference in the system can be minimized and optimal rates can be achieved.

This assumption may be feasible in small networks where the channels change slowly. However, channels change constantly in mobile networks. Thus much work has focused on how to achieve good system performance with reduced feedback. One transmission scheme uses limited CSI, where nodes feed back their CSI only to nearby nodes [1]. This approach assumes that the nearby nodes dominate the interference, and should therefore yield near-optimal performance. However, even in point-to-point systems, low channel coherence times make full CSI feedback infeasible. Many schemes thus utilize quantized forms of CSI to reduce feedback [2]. However, these approaches still suffer from having to feed back information every time the channel changes.

An alternate approach is to utilize the *channel statistics*, or Channel Distribution Information (CDI), to enhance communication. Because it takes into account the randomness in the channel, CDI is more robust to small channel coherence times and is thus valid for much longer than CSI. In addition, statistical data based on node location can also be collected and stored *a priori*, eliminating the need for real-time channel feedback. The ergodic capacity-optimal input covariance for the SU-MIMO channel with mean and covariance channel information is presented in [3]. This result is extended to the MIMO multiple access channel in [4]. Suboptimal solutions for the MIMO broadcast channel are given in [5]. These works focus on average capacity, but in doing so, do not account for a fundamental issue with using CDI: much of the time the throughput will be significantly less than the average due to varying channel conditions. Thus, previous schemes do not provide robust transmission over time.

This work seeks to address this problem by looking at the *outage* of the links in the network. Due to the randomness in the channel, reliable transmission cannot be achieved all the time using just CDI. However, in this work we guarantee a certain signal-to-interference-plus-noise ratio (SINR), and thus reliable transmission, for a specified probability on all the links using power control and linear beamforming. While many works in the literature assume the channels experience independent identically-distributed (i.i.d.) Rayleigh fading for analytic convenience, the Kronecker model captures correlation between the antennas and is a more accurate representation of what happens in real systems [6]. With the assumption of the Kronecker model on all the links, the expression for the outage probability on each link is derived. Then, we derive the optimal transmit and receive beamformers given this model and give an optimal power control algorithm. In Section 2, the problem formulation is given, and the expression for outage probability is derived. Section 3 derives the optimal beamformers and gives the power control algorithm. We plot our derived CDI solution for different outage thresholds in Section 4, and we summarize our results in Section 5.

In this work, the following notation is used: italicized letters indicate scalars (e.g. p_l), lower-case bold letters indicate vectors (e.g. $\mathbf{v}$), and upper-case bold letters indicate matrices (e.g. $\mathbf{A}$). Furthermore, $(\bullet)^H$ indicates the Hermitian operator, and $(\bullet)^*$ indicates the conjugate transpose operator.

2. PROBLEM FORMULATION

2.1. System Model

This work considers time-varying MIMO channels for many users in a network. Consider a MIMO network with L transmit-receive pairs. At link l, the transmitter sends the symbol $x_l(t)$ to the receiver. The transmitter uses unit-norm beamforming vector $\mathbf{v}_l(t)$ to precode the signal, and transmits with power $p_l(t)$. The receiver employs the linear unit-norm beamformer $\mathbf{u}_l(t)$ to combine the signal. The channel from transmitter i to receiver l is given by $\mathbf{H}_{li}(t)$. The noise $N_l(t)$ is distributed as a complex circular Gaussian, and represents the combined noise after applying the receive beamforming vector to the incoming signal.

In schemes that use perfect CSI, for every change in $\mathbf{H}_{li}(t)$, all the transmit and receive beamformers must be updated, and the

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power allocation scheme changes. This work will consider the case where the channel varies, but the statistics stay constant. Therefore the power allocation and transmit and receive beamformers stay fixed, and the time variable will be dropped for notational convenience. To further simplify notation, define $G_{li} = |\mathbf{u}_l^H \mathbf{H}_{li} \mathbf{v}_i|^2$ as the beamforming channel gain from the transmitter on link i to the receiver at link l and $\sigma^2_{N_l}$ as the noise power for the l'th link. Then, under this model, the SINR Γ_l on each link can be shown to be

$$\Gamma_l = \frac{p_l G_{ll}}{\sum_{i \neq l} p_i G_{li} + \sigma^2_{N_l}} \tag{1}$$

If perfect CSI is available, to ensure a reliable link is available to all nodes in the network, each link has an SINR constraint: Γ_l must be greater than a threshold γ_l. The goal is then to minimize the power consumed by the network while meeting all the SINR constraints [1]. When only CDI is available, however, the $\mathbf{H}_{li}$'s are unknown. Instead, they are assumed to be a random variable drawn from a complex-normal distribution, with a Kronecker structure on their correlation. The Kronecker model assumes the transmit and receive correlations are independent at the link ends. *This implies that each transmitter has its own transmit correlation matrix, regardless of the receiver, and each receiver has its own receive correlation matrix, regardless of the transmitter.* Thus, the channel from transmitter i to receiver l is given as:

$$\text{vec}(\mathbf{H}_{li}) \sim CN(0, \mathbf{\Sigma}^T_{T_i} \otimes \mathbf{\Sigma}_{R_l})$$

The knowledge of the transmit and receive correlation matrices comprise the CDI of the network. Under this model, the SINR becomes a random variable since it depends on the channel. Therefore, the constraints become outage constraints, so links are allowed to have an SINR below their thresholds γ_l for some probability α_l. The cost function considered in this work is the *weighted sum power*. In this setup, each link l in the network incurs some cost $w_l > 0$ to transmit across its link.

To simplify notation, define the weighting and power vectors as $\mathbf{w} = \{w_1, \ldots, w_L\}$ and $\mathbf{p} = \{p_1, \ldots, p_L\}$, respectively. The beamforming matrices are defined as $\mathbf{U} = \{\mathbf{u}_1, \ldots, \mathbf{u}_L\}$ and $\mathbf{V} = \{\mathbf{v}_1, \ldots, \mathbf{v}_L\}$. The main optimization problem for using CDI can then be formulated:

$$\begin{aligned} &\min_{\mathbf{p} \geq 0, \mathbf{U}, \mathbf{V}} \mathbf{w}^T \mathbf{p} \\ &\text{s.t.} \quad \Pr(\Gamma_l \leq \gamma_l) \leq \alpha_l, l = 1, \ldots, L \end{aligned} \tag{2}$$

To solve this problem, a closed-form expression for the outage probability, $\Pr(\Gamma_l \leq \gamma_l)$, must first be derived.

2.2. Derivation of Outage Probability

In order to get a closed-form expression for the outage probability, Γ_l is first expressed as a ratio of an exponential random variable to a weighted sum of exponential random variables plus some constant. The CDF for Γ_l is then given in [7], which gives the final expression for the outage probability.

First, the following Lemma is needed:

Lemma 1 *If* $\mathbf{u}$ *and* $\mathbf{v}$ *are unit-norm vectors and* $\mathbf{H}$ *is Gaussian matrix distributed as vec*$(\mathbf{H}) \sim CN(0, \mathbf{\Sigma}^T_T \otimes \mathbf{\Sigma}_R)$*, then*

$$\frac{|\mathbf{u}^H \mathbf{H} \mathbf{v}|^2}{(\mathbf{v}^H \mathbf{\Sigma}_T \mathbf{v})(\mathbf{u}^H \mathbf{\Sigma}_R \mathbf{u})} \sim \chi^2_2 \tag{3}$$

Proof: First, consider $Z = \mathbf{u}^H \mathbf{H} \mathbf{v}$. Z is a linear combination of 0-mean complex Gaussians, so Z is 0-mean complex circular Gaussian. The next step is to find the variance. To do this, perform the following calculations:

$$\begin{aligned} var(Z) &= E[Z \cdot Z^*] = E[(\mathbf{u}^H \mathbf{H} \mathbf{v})(\mathbf{u}^H \mathbf{H} \mathbf{v})^*] \\ &= E[\text{vec}(\mathbf{u}^H \mathbf{H} \mathbf{v}) \text{vec}(\mathbf{u}^H \mathbf{H} \mathbf{v})^H] \\ &= E[(\mathbf{v}^T \otimes \mathbf{u}^H) \text{vec}(\mathbf{H}) \text{vec}(\mathbf{H})^H (\mathbf{v}^* \otimes \mathbf{u})] \\ &= (\mathbf{v}^H \mathbf{\Sigma}_T \mathbf{v})(\mathbf{u}^H \mathbf{\Sigma}_R \mathbf{u}) \end{aligned}$$

This gives an expression for $var(Z)$. The random variable of interest, however, is $|Z|^2$. The norm squared of a unit variance complex circular Gaussian has a χ^2_2, or exponential, distribution. Then $|Z|^2$ needs to be normalized appropriately, which can be done by dividing by the variance, giving the result.

□

To simplify notation, define $t_l = (\mathbf{v}_l^H \mathbf{\Sigma}_{T_l} \mathbf{v}_l)$ and $r_l = (\mathbf{u}_l^H \mathbf{\Sigma}_{R_l} \mathbf{u}_l)$, $l = 1, \ldots, L$. Without loss of generality, the expression for outage probability of the first user will be shown. The theorem can then be stated as follows:

Theorem 1 *In a MU-MIMO network where all links experience correlated Rayleigh fading under the Kronecker model and the transmitter and receiver both employ linear beamforming, the expression for outage probability for the SINR is given by*

$$\rho_{out} = \Pr(\Gamma_1 \leq \gamma_1) = 1 - e^{\frac{\gamma \sigma^2_{N_1}}{2 r_1 t_1 p_1}} \prod_{i=2}^{L} \left(1 + \gamma_1 \frac{t_i p_i}{t_1 p_1}\right) \tag{4}$$

Proof: To apply the results from [7], ρ_{out} needs to be in the following form:

$$\rho_{out} = \Pr(\Gamma_1 \leq \gamma_1) = \Pr(\frac{X}{Y + \sigma^2} \leq \gamma_1), \tag{5}$$

where X is an exponential random variable and Y is a weighted sum of independent exponential random variables. Applying *Lemma 1* to the G_{li} terms in the expression for Γ_1 in Eqn. (1) gives

$$\begin{aligned} \Gamma_1 &= \frac{r_1 t_1 p_1 Z_1}{\sum_{i=2}^{L} r_1 t_i p_i Z_i + \sigma^2_{N_1}}, \\ Z_i &\sim \chi^2_2, Z_i's \text{ i.i.d.} \end{aligned}$$

Now, divide the top and bottom of the right hand side of the equation by $r_1 t_1 p_1$ and define $k_i = \frac{t_i p_i}{t_1 p_1}$, $X = Z_1$, $Y = \sum_{i=2}^{L} k_i Z_i$, and $\sigma^2 = \frac{\sigma^2_{N_1}}{r_1 t_1 p_1}$. After plugging in these variables into the expression for Γ_1, ρ_{out} has exactly the desired form of Eqn. (5). The closed-form expression given in [7] can then be applied to (5), which gives the final result for ρ_{out} above.

□

To apply this result to (2), first define $\beta_l = (1 - \alpha_l)^{-1}$. Then, manipulating the constraints gives the following optimization problem:

$$\min_{\mathbf{p}\geq 0,\mathbf{U},\mathbf{V}} \mathbf{w}^T\mathbf{p} \tag{6}$$
$$\text{s.t.} \quad e^{\frac{\gamma_l \sigma_{N_l}^2}{2r_l t_l p_l}} \prod_{i\neq l}\left(1+\gamma_l \frac{t_i p_i}{t_l p_l}\right) \leq \beta_l, l=1,\dots,L$$

This expression can also be generalized to channels with general correlation structures (without the Kronecker model restriction), and is derived and discussed in [8].

3. DERIVATION OF THE OPTIMAL SOLUTION

3.1. Optimal Transmit and Receive Beamforming

First, focus on the optimal transmit beamformers. Define $q_l = t_l p_l, l = 1,\dots,L$. By noting that $p_l = \frac{q_l}{t_l}$, the following equivalent optimization problem to (6) is presented:

$$\min_{\mathbf{q}\geq 0,\mathbf{U},\mathbf{V}} \sum_{l=1}^{L} \frac{w_l q_l}{t_l} \tag{7}$$
$$\text{s.t.} \quad e^{\frac{\gamma_l \sigma_{N_l}^2}{2r_l q_l}} \prod_{i\neq l}\left(1+\gamma_l \frac{q_i}{q_l}\right) \leq \beta_l, l=1,\dots,L$$

In the above optimization problem, the t_l's only appear in the optimization function, and not in the constraints. To minimize the objective function in (7), then, the t_l terms should be maximized. Since t_l is a function of $\mathbf{v}_l$, maximizing t_l is equivalent to solving the following optimization problem:

$$\max_{\mathbf{v}_l} \mathbf{v}_l^H \mathbf{\Sigma}_{T_l} \mathbf{v}_l \tag{8}$$
$$\text{s.t.} \quad ||\mathbf{v}_l||_2 = 1$$

This optimization problem has a well-known solution: $\mathbf{v}_l$ is the normalized eigenvector corresponding to the maximum eigenvalue of $\mathbf{\Sigma}_{T_l}, l = 1,\dots,L$. Thus, this gives the optimal $\mathbf{V}$ to solve (7). Since the problem in (7) is equivalent to the problem in (6), the optimal transmit beamformers $\mathbf{V}$ will be the same as well.

Now focus on the optimal receive beamformers. In (6), note that r_l only appears in the exponential term, $e^{\frac{\gamma_l \sigma_{N_l}^2}{2r_l t_l p_l}}$, of the l^{th} constraint equation. The objective function will be minimized when the p_l's are as small as possible while still meeting the constraints. In order for p_l to be minimized while meeting the l^{th} constraint, observe that the exponential term with respect to r_l should be minimized. Since r_l appears in the denominator of the exponential term, this term will be minimized when r_l is maximized. Thus, the optimal receive beamformer for the l^{th} constraint will be the $\mathbf{u}_l$ that maximizes r_l. This is analogous to the optimal transmit beamformer case, so $\mathbf{u}_l$ is the normalized eigenvector corresponding to the maximum eigenvalue of $\mathbf{\Sigma}_{R_l}, l = 1,\dots,L$. This gives the optimal receive beamformers $\mathbf{U}$.

3.2. Power Control

In the following analysis, the transmit and receive beamformers are assumed to be fixed. It can be shown that if all the beamformers are fixed, the constraints in (6) are satisfied with equality and thus uniquely determine the optimal power vector for all choices of $\mathbf{w}$ in the optimal solution [9]. Then, to find the optimal $\mathbf{p}$, first take the logarithm of the constraint equations in (6) and multiply both sides by $\frac{p_l}{\log \beta_l}$. This results in

$$\frac{\gamma_l \sigma_{N_l}^2}{2r_l t_l \log \beta_l} + \frac{p_l}{\log \beta_l} \sum_{i\neq l} \log\left(1+\gamma_l \frac{t_i p_i}{t_l p_l}\right) \leq p_l$$

Now define

$$I_l(\mathbf{p}) = \frac{\gamma_l \sigma_{N_l}^2}{2r_l t_l \log \beta_l} + \frac{p_l}{\log \beta_l} \sum_{i\neq l} \log\left(1+\gamma_l \frac{t_i p_i}{t_l p_l}\right)$$
$$\mathbf{I}(\mathbf{p}) = [I_1(\mathbf{p}),\dots,I_L(\mathbf{p})]$$

The function $\mathbf{I}(\mathbf{p})$ is identical to the function presented in [9], with different constants, and in their work is shown to be a *standard interference function*. A key property of standard interference functions is that they satisfy $\mathbf{p} \geq \mathbf{I}(\mathbf{p})$. Thus, using this function and a starting power vector $\mathbf{p}$, the update equation for the algorithm is given as:

$$\mathbf{p}^{(n+1)} = \mathbf{I}(\mathbf{p}^{(n)}) \tag{9}$$

The complete algorithm can then summarized as follows:

Algorithm 1: Joint Power Control and Beamforming

1. Solve for the optimal $\mathbf{u}_l$'s, and $\mathbf{v}_l$'s, $l = 1,\dots,L$, as the normalized dominant eigenvectors of $\mathbf{\Sigma}_{R_l}$'s and $\mathbf{\Sigma}'_{T_l}s$, respectively, as shown in Section 3.1
2. Initialize $\mathbf{p} \geq 0$ and update $\mathbf{p}$ using (9) until convergence

In the special case of SU-MIMO (where $L = 1$), the optimal beamformers have the same solution as in the MU-MIMO case, and the optimal power p can be given in closed form:

$$p_{opt} = \frac{\gamma \sigma_N^2}{2(\mathbf{v}_{opt}^H \mathbf{\Sigma}_T \mathbf{v}_{opt})(\mathbf{u}_{opt}^H \mathbf{\Sigma}_R \mathbf{u}_{opt}) \log(\beta)} \tag{10}$$

4. RESULTS

We conduct some numerical experiments to understand the efficacy of the joint power control and beamforming algorithm developed above. Results are given for a single-user MIMO setup and a MU-MIMO setup. All transmitters and receivers have 4 antennas, and the SNR is held constant at 10 dB. Equal SINR thresholds and equal outage constraints are considered for all users in the system.

For single-user MIMO, an algorithm using perfect channel knowledge is compared to the algorithm presented here, which uses the covariance information only, in *Experiment 1*. The covariance matrix here uses the Kronecker model parameters extracted from a covariance matrix generated from an angular spread model using 100 scatterers with a transmit and receive angular spread of $\frac{\pi}{8}$ coming in at broadside. The algorithm using perfect channel knowledge uses the principal left and right singular vectors for receiver and transmitter beamforming, respectively, and calculates the minimum transmit power required to achieve the threshold (this is an optimal scheme). See Fig. 1.

The plot yields some interesting results: the average transmit powers for the covariance schemes are actually *lower* than for using the true channel at 5%, 10%, and 20% outage values. To calculate this, only the covariance information is required, so it needs to be updated only when the channel statistics change. Note also that the plots are in log scale, so the savings over true channel knowledge

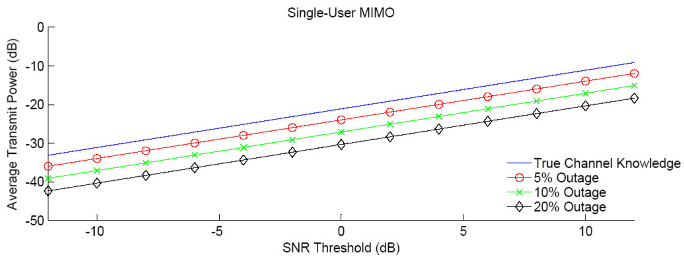

Fig. 1. Single User Case

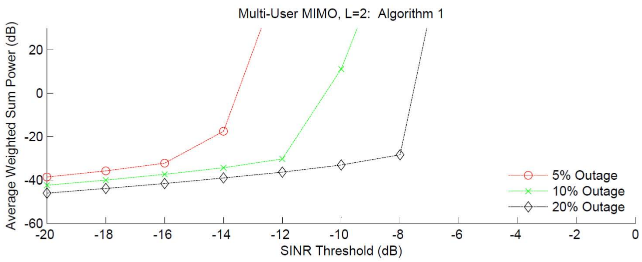

Fig. 2. Two-User Case

are significant; at 5% outage, there is roughly a 3dB gap, and at 10% outage there is roughly a 6dB gap. The tradeoff is that the covariance algorithm will allow the link to fail with the specified outage probability. However, this may be a desirable tradeoff for many applications since it prevents the transmitter from attempting to increase the power to compensate for poor channel conditions, thereby prolonging battery life.

For the MU-MIMO setup, a network with 2 transmit/receive links and a weighting vector $w = [10, 1]^T$ is considered. The covariance matrices use the Kronecker model parameters extracted from a covariance matrix generated from an angular spread model using 100 scatterers with a transmit angular spread of $\frac{\pi}{16}$ and a receive angular spread of $\frac{2\pi}{3}$. For link 1, the beams are centered at broadside, and for link 2, the beams are centered at the null. In *Experiment 2*, different outage thresholds obtained from **Algorithm 1** are compared. See Fig. 2.

As expected, this experiment shows that as the outage probability threshold is increased, the power required by the network to meet the threshold also increases. Also, for the multiuser case, the solutions generated by **Algorithm 1** become infeasible (when the curves go to ∞) as SINR requirement increases. The lower the outage, the lower the SINR threshold has to be for the outage to become infeasible. The SINR thresholds here are low due to the limitations of the Kronecker model and the inability of the optimal beamformers to suppress interference, as discussed in Section 3.

5. CONCLUSION

In this work, a framework has been presented for analyzing MU-MIMO networks when only the Kronecker model parameters are known. The expression for outage probability was derived in closed-form, and the optimal solution for the beamformers and power control was given. When compared to CSI, the computational complexity for the algorithm and feedback information required for covariance information is drastically reduced since both must be done only once for valid covariance information. Also, in the single-user case, the amount of power used at the receiver is reduced when compared to the optimal algorithm using CSI on average.

The assumption of the Kronecker model in the multiuser case is not realistic in many scenarios. The scatterers between one receiver and transmitter is likely to differ from another receiver and the same transmitter, and so the correlation structures can differ at both the receive and transmit side, even when considering the same transmitter. Much work has been done in developing more accurate models for real systems (e.g. [10]). We address the problem of general correlation structures on every link in [8], where no specific correlation model is assumed. A jointly optimal solution is difficult to achieve, but the optimal power control algorithm presented here can be extended to the general case, and a very good suboptimal solution for the transmit and receive beamformers is also given. In the general case, the beamformers are able to intelligently cancel interference, and we see that savings can be achieved in power when compared to CSI algorithms.

6. REFERENCES

[1] B. Song et al., "Network Duality for Multiuser MIMO Beamforming Networks and Applications," *IEEE Trans. Commun.*, vol. 55, no. 3, pp. 618–630, Mar. 2007.

[2] M. Sharif and B. Hassibi, "On the Capacity of MIMO Broadcast Channels with Partial Side Information," *IEEE Trans. Inf. Theory*, vo. 41, no. 2, pp. 506–522, Feb. 2005.

[3] A.M. Tulino et al., "Capacity-Achieving Input Covariance for Single-User Multi-Antenna Channels," *IEEE Trans. Wireless Commun.*, vol. 5, no. 3, pp. 662–671, Mar. 2006.

[4] A. Soysal and S. Ulukus, "Optimum Power Allocation for Single-User MIMO and Multi-User MIMO-MAC with Partial CSI," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 7, pp.1402–1412, Sept. 2007.

[5] A.L. Anderson et al., "Reduced-Feedback Linear Precoding with Stable Performance for the Time-Varying MIMO Broadcast Channel," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 8, pp. 1483–1493, Oct. 2008.

[6] J.P. Kermoal et al., "A Stochastic MIMO Radio Channel Model with Experimental Validation," *IEEE J. Sel. Areas Commun.*, vol. 20, no. 6, pp. 1211–1226, Aug. 2002.

[7] A. Annamalai et al., "Simple and Accurate Methods for Outage Analysis in Cellular Mobile Radio Systems–A Unified Approach," *IEEE Trans. Commun.*, vol. 49, no. 2, pp. 303–316, Feb. 2001.

[8] S. Ghosh et al., "Outage-Efficient Strategies for Multiuser MIMO networks with Channel Distribution Information," *In Preparation*.

[9] J. Papandriopoulos et al., "Optimal Power Control for Rayleigh-Faded Multiuser Systems With Outage Constraints," *IEEE Trans. Wireless Commun.*, vol. 4, no. 6, pp. 2705–2715, Nov. 2005.

[10] W. Weichselberger, M. Herdin, H. Ozcelik, and E. Bonek, "A Stochastic MIMO Channel Model with Joint Correlation at Both Link Ends," *IEEE Transactions on Wireless Communications*, vol. 5, no. 1, pp. 90–100, Jan. 2006.